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A phenomenological model of ferromagnetic martensite

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Abstract. The ferromagnetism of the inhomogeneous crystalline structure arising below the temperature of the martensitic phase transition is considered using a phenomenological expression for the Helmholtz free energy of a cubic crystal. The magnetic anisotropy of the martensitic structure is related to the order parameter of the ferroelastic phase transition. The expressions for the static magnetic susceptibility and for the magnetization of the martensite are derived. The theoretical results are compared with the experimental data known for Ni_2MnGa alloy. The magnetoelastic constant and the constant of strain-induced magnetic anisotropy of this alloy are estimated from the experimental magnetic field dependence of the magnetization.

1. Introduction

A martensitic phase transition is associated with the spontaneous deformation of the crystal lattice below the transition temperature T_M. The high-temperature cubic phase (austenite) is spatially homogeneous, while the state arising at $T < T_M$ (martensite) is inhomogeneous. The spatial structure of martensite is governed by the condition of self-accommodation of the spontaneous strains. Some materials undergoing martensitic transitions are ferromagnets with Curie temperatures $T_C > T_M$, and, therefore, the ferromagnetic martensites appear below T_M. Ferromagnetic martensites have been observed among Fe- and Ni-based shape memory and superelastic alloys [1–8].

Martensitic transitions are accompanied by pronounced anomalies of the magnetic properties of the ferromagnets. In particular, sharp changes of the spontaneous magnetization, magnetic susceptibility [1–3, 5, 6], and magnetostriction [7, 8] were observed in the vicinity of the phase transition temperature T_M. These effects are of interest in view of the possible applications, e.g. in devices utilizing the large strains induced by the magnetic field [7].

In the present paper we intend to develop a phenomenological model of a ferromagnetic martensite and to apply this model to the stoichiometric and nonstoichiometric Ni_2MnGa Heusler alloys with $T_M = 202$ K, $T_C = 376$ K [1] and $T_M = 285$ K, $T_C = 375.5$ K [5], respectively.

The intermediate phase (the precursor) is observed in Ni_2MnGa alloy at $T_M < T < T_1 = 260$ K [9–12]. The periodic displacements of the atoms from the initial positions

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in the cubic crystal lattice arise at $T = T_1$, but the average strains stay equal to zero in the intermediate phase [11]. The transition to the intermediate phase is accompanied by a softening of the phonon spectra [9–11] and elastic moduli [13]. By contrast, the anomalies of the spontaneous magnetization and magnetic susceptibility were not observed in the vicinity of the intermediate phase transition [1, 3]. This means that the magnetization of the alloy is not sensitive to the appearance of precursor phenomena [11] but is strongly influenced by the spontaneous strains arising at $T = T_M$ [3].

The proposed theoretical model relates the magnetic anomalies accompanying cubic–tetragonal martensitic transitions to the spatially inhomogeneous strains arising at $T = T_M$. The model is based on Landau expansions used by many authors for the description of both elastic [14] and magnetic [4] subsystems of martensitic alloys (see also the review paper [15]). After obvious generalization, this model may be applied to the description of twinned magnetically ordered crystals.

2. The Helmholtz free energy of ferromagnetic martensite

The Helmholtz free energy of a cubic ferromagnet may be written down as the sum

$$F = F_e + F_m + F_{me}. \tag{1}$$

The first term of the sum describes the strain energy and may be expressed as [16]

$$F_e = \frac{3}{2}(C_{11} + 2C_{12})u_1^2 + \frac{1}{6}C'(u_2^2 + u_3^2) + \frac{1}{2}C_{44}(u_4^2 + u_5^2 + u_6^2) + \frac{1}{3}au_3(u_3^2 - 3u_2^2) + \frac{1}{4}b(u_2^2 + u_3^2)^2 \tag{2}$$

where C_{11}, C_{12}, C_{44}, and $C' = (C_{11} - C_{12})/2$ are the second-order elastic moduli of the crystal, a and b are the linear combinations of third- and fourth-order elastic moduli respectively, and $u_1, u_2, \ldots, u_6$ are linear combinations of strain tensor components:

$$\begin{aligned} u_1 &= (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})/3 \\ u_2 &= \sqrt{3}(\varepsilon_{xx} - \varepsilon_{yy}) \\ u_3 &= 2\varepsilon_{zz} - \varepsilon_{yy} - \varepsilon_{xx} \\ u_4 &= \varepsilon_{yz} \\ u_5 &= \varepsilon_{xz} \\ u_6 &= \varepsilon_{xy}. \end{aligned} \tag{3}$$

The third-order and fourth-order terms depending on u_4, u_5, and u_6 are immaterial as regards the further analysis, and, therefore, are omitted in (2).

The second term in (1) may be chosen in the simplest form:

$$\begin{aligned} F_m &= F_A - \boldsymbol{M} \cdot \boldsymbol{H} \\ F_A &= K(m_z^2 m_y^2 + m_z^2 m_x^2 + m_y^2 m_x^2) \end{aligned} \tag{4}$$

where $\boldsymbol{m} = \boldsymbol{M}/|\boldsymbol{M}|$, $\boldsymbol{M}$ is the magnetization of the crystal, $\boldsymbol{H}$ is the external magnetic field, and K is the magnetic anisotropy constant.

The last term in (1) describes the interrelation between the magnetization and the strains:

$$F_{me} = -\delta_1\left[(2m_z^2 - m_y^2 - m_x^2)u_3 + \sqrt{3}(m_x^2 - m_y^2)u_2\right] - \delta_2(m_z m_y u_4 + m_z m_x u_5 + m_y m_x u_6) \tag{5}$$

where δ_1 and δ_2 are the magnetoelastic constants.

The isotropic parts of the energies F_m and F_{me} are immaterial, and therefore are disregarded in (4) and (5).

The coefficients of the Landau expansion for the free energy of Fe_3Ni alloy were evaluated 'from first principles' in [4], but the terms (5) describing the linear coupling of m_i^2 and u_β ($\beta = 2, 3, \ldots, 6$) were disregarded.

In the absence of mechanical stresses, the conditions $\partial F/\partial u_\beta = 0$ are fulfilled, and the strains of the austenite are caused only by the magnetization of the crystal:

$$\begin{aligned} u_2^{me} &= (3\sqrt{3}\delta_1/C')(m_x^2 - m_y^2) \\ u_3^{me} &= (3\delta_1/C')(2m_z^2 - m_y^2 - m_x^2) \\ u_4^{me} &= (\delta_2/C_{44})m_y m_z \\ u_5^{me} &= (\delta_2/C_{44})m_x m_z \\ u_6^{me} &= (\delta_2/C_{44})m_x m_y. \end{aligned} \tag{6}$$

Below the temperature of the martensitic phase transition

$$u_\beta = u_\beta^{me} + u_\beta^M \tag{7}$$

where the u_β^M are caused by the strains attributed to the equilibrium values of the order parameter of the transition.

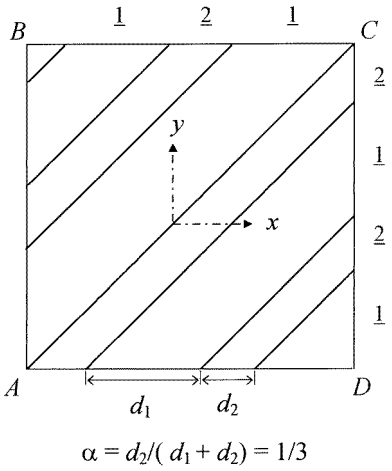

Figure 1. A schematic representation of the magnetic domain ABCD occupying the martensite variant formed by the crystallographic domains 1 and 2.

The cubic–tetragonal phase transition with the two-component order parameter (u_2, u_3) is a frequently encountered kind of martensitic transition [17]. Let us consider the martensite variant, which is the periodic structure formed by alternating domains of the tetragonal phase (figure 1). Let $\boldsymbol{x}$ and $\boldsymbol{y}$ be parallel to the fourfold symmetry axes of neighbouring domains. The averaged strains $\bar{\varepsilon}_{ij}$ peculiar to such martensitic structure have been analysed by many authors (see, for example, [18, 19]). Here it is convenient to use the formulae derived in [20], because these formulae express the strains in terms of the order parameter in the

explicit form

$$\begin{aligned}
\bar{\varepsilon}_{xx} &= \frac{1}{6}(3\alpha - 1)u_0 - \frac{1}{4}\alpha(1-\alpha)u_0^2 \\
\bar{\varepsilon}_{yy} &= \frac{1}{6}(2 - 3\alpha)u_0 - \frac{1}{4}\alpha(1-\alpha)u_0^2 \\
\bar{\varepsilon}_{zz} &= -\frac{1}{6}u_0 \\
\bar{\varepsilon}_{xy} &= \frac{1}{4}\alpha(1-\alpha)u_0^2 \\
\bar{\varepsilon}_{xz} &= \bar{\varepsilon}_{yz} = 0
\end{aligned} \tag{8}$$

where α and $1-\alpha$ are the volume fractions of domains, and u_0 is related to the equilibrium value (u_2^0, u_3^0) of the order parameter:

$$u_0 = \pm 2u_2^0/\sqrt{3} = -2u_3^0 = -(a/2b)[1 + (1 - 4bC'/3a^2)^{1/2}].$$

(The plus/minus sign corresponds to the x-domain/y-domain.) The expression for u_0 was obtained on the assumption that $u_\beta^{me} \ll u_\beta^M$, $3au_\beta^{me}/C' \ll 1$, and $2bu_\beta^{me}/a \ll 1$. As far as we are aware, these conditions are fulfilled for all of the alloys (e.g. for Ni_2MnGa, $u_\beta^{me} \sim 10^{-4}$–$10^{-5}$ [7], $u_\beta^M \simeq 5 \times 10^{-2}$ [21]). The ratios of the elastic moduli may be estimated as $a/C' \sim 10^1$–10^2, $b/a \sim 1$ [22].

It is of importance that the averaged strain tensor (8) describing the martensite variant has monoclinic symmetry, while the symmetry of (u_2^0, u_3^0) is tetragonal (for more details see [19, 20]).

The value of α is prescribed by the type of periodic structure. For definiteness, consider the values $\alpha = 1/3$ and $\alpha = 2/3$ which are inherent to the frequently encountered R9 structure. The strains (8) must now be substituted into (3) and (7). As long as the inequality $u_0 \ll 1$ holds for the martensitic phase transitions, the averaged strains accompanying the appearance of the R9 structure with $\alpha = 1/3$ may be approximated by the formula $u_3^M = \sqrt{3}u_2^M \approx -u_0/2$. In this approximation the averaged strain tensor has orthorhombic symmetry ($\bar{\varepsilon}_{xy} \approx 0$). Substitution of u_2^M and u_3^M into (4) and (5) results in the expressions

$$\begin{aligned}
F_m + F_{me} &= \tilde{F}_A + F^{(yz)} \\
\tilde{F}_A &= \tilde{K}(m_z^2 m_y^2 + m_z^2 m_x^2 + m_y^2 m_x^2) \\
F^{(yz)} &= \delta_1 u_0 (m_z^2 - m_y^2) - \boldsymbol{m} \cdot \boldsymbol{H} M_0
\end{aligned} \tag{9}$$

where

$$\tilde{K} = K + (18\delta_1^2/C') - (\delta_2^2/2C_{44}) \qquad M_0 \equiv |\boldsymbol{M}|.$$

(The obvious condition $m^2 = 1$ was taken into account and the constant summand in $\tilde{F}_A$ was omitted.)

The energy

$$F^{(xz)} = \delta_1 u_0 (m_z^2 - m_x^2) - \boldsymbol{m} \cdot \boldsymbol{H} M_0 \tag{10}$$

corresponds to the value $\alpha = 2/3$.

It is evident now that the energy of each variant of the R9 structure may be expressed in the form

$$\begin{aligned}
F &= \tilde{F}_A + F^{(ij)} \\
F^{(ij)} &= -A(m_i^2 - m_j^2) - \boldsymbol{m} \cdot \boldsymbol{H} M_0
\end{aligned} \tag{11}$$

where the constant $A = \delta_1 u_0$ describes the magnetic anisotropy induced by the averaged strains, $i, j = x, y, z, i \neq j$.

For definiteness, we take $A > 0$, and, therefore, subscript i marks the 'easy axis' for magnetization and j corresponds to the 'hard axis'.

The magnetic anisotropy of each martensitic structure, whose symmetry is close to orthorhombic, may be described by the expression

$$F = A_1 m_i^2 + A_2 m_j^2. \tag{12}$$

The concrete type of the structure indicates the dependences $A_{1,2}(\delta_1, \delta_2, u_0)$.

3. Magnetization and magnetic susceptibility of martensite

It is common knowledge that the process of magnetization of a ferromagnetic specimen is accompanied by the shifting of the magnetic domain walls, and by the rotation of the vector $\boldsymbol{m}$. As was pointed out in [23], the motion of magnetic domain walls may be hindered by the inhomogeneous structure of martensite, and the rotation of $\boldsymbol{m}$ may be the dominant mechanism of magnetization of the specimen. In the case under consideration, this viewpoint can be substantiated in the following way.

Let the variant of martensite with the easy axis x border on the variant with the easy axis y. In zero magnetic field the magnetic moment of the first variant is directed along $\boldsymbol{x}$ and the moment of the second one is parallel to $\boldsymbol{y}$. Under such conditions, these variants are equivalent magnetic domains. In a strong magnetic field applied along $\boldsymbol{x}$, both magnetic moments are aligned along $\boldsymbol{x}$. Now the energy of the first variant differs from the energy of the second one by $\Delta F = 2A$, and, therefore, the variants represent two different magnetic phases. The first one is stable for $0 < H_x < \infty$, while the second one is stabilized for $H_x > H_1$, where H_1 is the field of the spin-orientational phase transition. Such transitions are usually treated as a rotation of the vector $\boldsymbol{m}$ from the y- to the x-direction.

If one martensite variant is occupied by two ferromagnetic domains with antiparallel moments, the wall separating these domains can move easily, and one of the domains will disappear at some magnetic field value H_c of the order of the coercive force of the specimen. It may be expected, hence, that within the interval $H_c < H < H_1$ every variant will be occupied by the only magnetic domain, and the magnetic moment of the domain will rotate under the action of the magnetic field. Experimental data obtained in [8] and [7] allow us to conclude that for Ni_2MnGa alloy the field H_1 is at least 20 times greater than the coercive force (see section 4).

For the sake of simplicity, assume that $\tilde{K} \ll A$. (This assumption will be justified in section 4 for the Ni_2MnGa crystal, which typifies ferromagnetic martensites.) In such a case, the energy $\tilde{F}_A$ may be disregarded and the angle ψ between the stationary magnetic field and vector the $\boldsymbol{m}$ of the variant may be found from the condition

$$\frac{\partial F^{(ij)}}{\partial \psi} = 0. \tag{13}$$

If the hard axis of the variant is parallel to the applied field, the energy (11) can be expressed as

$$F^{(ij)} = A(\cos^2 \psi - \sin^2 \psi) - H M_0 \cos \psi \tag{14}$$

and the condition (13) yields

$$\cos \psi \equiv \cos \psi_1 = \begin{cases} H/H_1 & \text{for } H < H_1 \\ 1 & \text{for } H \geqslant H_1 \end{cases} \tag{15}$$

where $H_1 = 4A/M_0$.

When both the easy and hard axes of the variant are perpendicular to the magnetic field,

$$F^{(ij)} = -A\sin^2\psi - HM_0\cos\psi \tag{16}$$

$$\cos\psi \equiv \cos\psi_2 = \begin{cases} H/H_2 & \text{for } H < H_2 \\ 1 & \text{for } H \geqslant H_2 \end{cases} \tag{17}$$

where $H_2 = 2A/M_0$.

Finally, if the easy axis is directed along the field direction,

$$\psi \equiv \psi_3 = \begin{cases} 0, \pi & \text{for } H < H_c \\ 0 & \text{for } H > H_c. \end{cases} \tag{18}$$

Generally speaking, in a field parallel to the [100] direction of a cubic crystal, all of the possibilities mentioned above are equally likely. At $H > H_c$ the averaged magnetization of a martensitic phase may be approximated by the formula

$$\langle \boldsymbol{M} \rangle = \frac{1}{3} M_0(T)(1 + \cos\psi_1 + \cos\psi_2)\boldsymbol{n} \tag{19}$$

where $\boldsymbol{n}$ is the unit vector in the direction of the field.

The magnetic susceptibility $\langle\chi\rangle$ of a martensite may be obtained in a routine way from (11). At $\tilde{K} \ll A$ the resultant formula is

$$\langle\chi\rangle = \frac{1}{6}\left[\chi^{(zy)} + \chi^{(yz)} + \chi^{(zx)} + \chi^{(xz)} + \chi^{(yx)} + \chi^{(xy)}\right] \tag{20}$$

where the $\chi^{(ij)}$ are the susceptibilities of the variants with the energies $F^{(ij)}$. When the stationary magnetic field is directed along $\boldsymbol{x}$,

$$\chi^{(zy)} = \begin{pmatrix} \chi_2 & 0 & \chi_8 \\ 0 & \chi_1 & 0 \\ \chi_8 & 0 & \chi_6 \end{pmatrix} \qquad \chi^{(yz)} = \begin{pmatrix} \chi_2 & \chi_8 & 0 \\ \chi_8 & \chi_6 & 0 \\ 0 & 0 & \chi_1 \end{pmatrix}$$

$$\chi^{(zx)} = \begin{pmatrix} \chi_1 & 0 & \chi_7 \\ 0 & \chi_2 & 0 \\ \chi_7 & 0 & \chi_5 \end{pmatrix} \qquad \chi^{(xz)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \chi_4 & 0 \\ 0 & 0 & \chi_3 \end{pmatrix} \tag{21}$$

$$\chi^{(yx)} = \begin{pmatrix} \chi_1 & \chi_7 & 0 \\ \chi_7 & \chi_5 & 0 \\ 0 & 0 & \chi_2 \end{pmatrix} \qquad \chi^{(xy)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \chi_3 & 0 \\ 0 & 0 & \chi_4 \end{pmatrix}$$

where

$$\chi_1 = \chi_2/2 = M_0^2/(4A)$$
$$\chi_3 = \chi_1 M_0/(M_0 + \chi_1 H)$$
$$\chi_4 = \chi_2 M_0/(M_0 + \chi_2 H)$$
$$\chi_5 = \chi_1 \operatorname{cotan}^2\psi_1$$
$$\chi_6 = \chi_2 \operatorname{cotan}^2\psi_2$$
$$\chi_7 = -\chi_1 \operatorname{cotan}\psi_1$$
$$\chi_8 = -\chi_2 \operatorname{cotan}\psi_2.$$

The expressions for $\chi^{(zx)}$ and $\chi^{(yx)}$ are valid while $H < H_1$, and the formulae for $\chi^{(zy)}$ and $\chi^{(yz)}$ are valid when $H < H_2$.

At $H \geqslant H_1$,

$$\chi_{zz}^{(zx)} = \chi_{yy}^{(yx)} = \chi_1 M_0/(\chi_1 H - M_0)$$
$$\chi_{yy}^{(zx)} = \chi_{zz}^{(yx)} = \chi_2 M_0/(\chi_2 H - M_0).$$

At $H \geqslant H_2$,

$$\chi_{zz}^{(zy)} = \chi_{yy}^{(yz)} = \chi_2 M_0/(\chi_2 H - M_0)$$
$$\chi_{zz}^{(yz)} = \chi_{yy}^{(zy)} = \chi_1 M_0/(\chi_1 H + M_0/2).$$

All of the other components of these tensors are equal to zero in the high-field range.

It is worth recalling that the expressions (19)–(21) were obtained for the R9 structure assuming that $A > 0$ and $H > H_c$, where H_c is of the order of the coercive force of the martensite. These expressions are directly applicable only to a single crystal in a magnetic field parallel to the [100] direction. Nevertheless, it will be shown later that the formula (19) agrees semi-quantitatively with the experimental data obtained for both single crystals and polycrystals of Ni_2MnGa ferromagnet. On making the change $A \to -A$, the expressions (14)–(21) became valid for $A = \delta_1 u_0 < 0$.

The other martensitic structures may easily be considered with the help of the energies (12) and strains (8). The consideration results in the dependencies (15), (17), and (19) with $H_1 = 2|A_1 - A_2|/M_0$, and $H_2 = 2|\acute{A}|/M_0$, where $\acute{A} \equiv \min\{A_1, A_2\}$.

In the case of R7 structure with $\alpha = 2/7$ or $5/7$, $A_1 = 6\delta_1 u_0/7$ and $A_2 = -9\delta_1 u_0/7$. For the five-layer structure with $\alpha = 1/5$ or $4/5$, the values of the anisotropy constants are $A_1 = 3\delta_1 u_0/5$ and $A_2 = -9\delta_1 u_0/5$.

So, the periodic martensitic structure may be characterized by the ratio H_1/H_2: for the R9 structure $H_1/H_2 = 2$; for the R7 structure $H_1/H_2 = 5/2$ or $5/3$ (at $\delta_1 u_0 < 0$ and $\delta_1 u_0 > 0$ respectively); and for the five-layer structure $H_1/H_2 = 4$ or $4/3$ (at $\delta_1 u_0 < 0$ and $\delta_1 u_0 > 0$ respectively).

4. Analysis of existing experimental data

The results of phenomenological analysis may be compared with the experimental data obtained for Ni_2MnGa ferromagnet in [1, 5, 8].

The experimental $\langle \boldsymbol{M}(H) \rangle$ dependence saturates at $H > H_s \approx (8\text{–}12) \times 10^3$ Oe [8, 7]. In accordance with (15) and (17)–(19), we will take $H_s = H_1 = 2|A_1 - A_2|/M_0$. The experimental value $I(T_M) \equiv |\langle \boldsymbol{M}(T_M) \rangle|/\rho \approx 85$ G cm^3 g^{-1} [1] corresponds to the magnetization $M_0(T_M) \approx 690$ G which results in the following range of values for the magnetic anisotropy constants of martensite:

$$|A_1 - A_2| \approx (2.8\text{–}4.2) \times 10^6 \text{ erg cm}^{-3}. \tag{22}$$

($\rho \approx 8.1$ g cm^{-3} is the mass density of Ni_2MnGa alloy.)

As far as we are aware, the values of the anisotropy constants of nickel-based ferromagnets are of the order of 10^5 erg cm^{-3} or less (see, e.g., [23]). Therefore, the magnetic anisotropy of the martensitic phase of Ni_2MnGa seemingly exceeds that of the austenitic phase ($A_{1,2} \gg \tilde{K}$).

The longitudinal (with respect to the applied field) magnetostriction λ of a macroscopically isotropic specimen is expressed by the well-known formula

$$\lambda = \frac{1}{2}\lambda_s[3(\boldsymbol{m} \cdot \boldsymbol{n}) - 1] \tag{23}$$

where $\lambda_s = 2\delta_1/C'$.

At $H > H_s$, the vector $\boldsymbol{n}$ is parallel to $\boldsymbol{m}$, and the transverse magnetostriction $\lambda_\perp$ may be estimated as

$$\lambda_\perp \approx \lambda/2 = |A_1 - A_2|/(2C'u_0) \approx (2\text{–}3) \times 10^{-5} \tag{24}$$

($C' \approx 6 \times 10^{11}$ erg cm^{-3}, $u_0 \approx 0.12$ [24], $\delta_1 = |A_1 - A_2|/(2u_0) \sim 10^7$ erg cm^{-3}).

The experimental value $\lambda_\perp \approx 2.5 \times 10^{-5}$ [8] measured at the temperature $T \approx T_M$ belongs to the estimated interval (24). This is the main confirmation that the proposed theoretical model is reasonable.

The experimental dependences $I(T)$ were observed for the polycrystalline specimen at the field values $H = 1$, 4, 8, and 16 kOe [1] and for the single crystal at $H = 0.82$ and 15 kOe [5]. It is easy to compute the $I(T)$ dependences from (15), (17), and (19). The dependence $M_0(T)$ involved in (19) may be approximated by standard function

$$M_0(T) = M_0(0)y(T) \qquad y(T) = \tanh[T_C\, y(T)/T]. \tag{25}$$

Figures 2(a) and 2(b) show the results of computations for $I(0) = 92$ G cm^3 g^{-1}, $T_C = 376$ K and two values of the martensitic phase transition temperature: $T_M = 202$ K (for the stoichiometric Ni_2MnGa composition [1]) and $T_M = 285$ K (for the nonstoichiometric composition of the alloy [5]). Solid lines correspond to the five-layer martensitic structure, while dashed ones were computed for the 7R structure. Both structures were observed experimentally in Ni_2MnGa alloy [21]. The experimental points obtained in [1] and [5] are presented in figures 2(a) and 2(b) for comparison.

5. Conclusions

(i) The drastic changes in the magnetic properties of ferromagnets occurring in the vicinity of the martensitic phase transition may be satisfactorily described within the framework of simple phenomenological theory. As the theory shows, these changes are mainly caused by the inhomogeneous strains arising below the phase transition temperature.

(ii) For the Ni-based and Fe-based alloys, the strain-induced magnetic anisotropy of martensite significantly exceeds the anisotropy of cubic austenite. As a consequence, over a wide range of applied magnetic fields the inhomogeneous strains govern the process of magnetization of martensite. This, in turn, enables a theoretical estimation of the magnetoelastic constant to be made from the field dependence of the magnetization.

(iii) The estimated magnetostriction of Ni_2MnGa alloy is close to the value measured in [8]. The theoretical temperature dependences of the magnetization are in agreement with the experimental data [1] obtained for Ni_2MnGa alloy at $H = 4$, 8, and 16 kOe (see figure 2(a)). The disagreement of the theoretical curve with the magnetization values measured at $H = 1$ kOe for $T < T_M$ is seemingly caused by the polycrystalline structure of the specimen examined in [1]. The experimental $M(T)$ dependence observed in [5] for the single crystal at $H = 0.82$ kOe is comparatively close to the theoretical one (see figure 2(b)). In this case some disagreement between theory and experiment occurs for both intervals $T < T_M$ and $T > T_M$. This shortcoming of the theoretical $I(T)$ dependence is not surprising, because the field $H = 0.82$ kOe is smaller than the saturation fields of austenite ($H_{sm} \simeq 1.5$ kOe [7]) and martensite ($H_s = 8$–12 kOe [7, 8]), while the computations were carried out for the value $I(0) = 90$ G cm^3 g^{-1} corresponding to zero temperature and $H = 15$ kOe. Nevertheless, a theoretical jump of the magnetization at $T = T_M$, $H = 0.82$ kOe is equal to the experimental one.

The coincidence of the estimated value of the magnetostriction of the Ni_2MnGa alloy with the value measured in [8] confirms that the proposed phenomenological model is

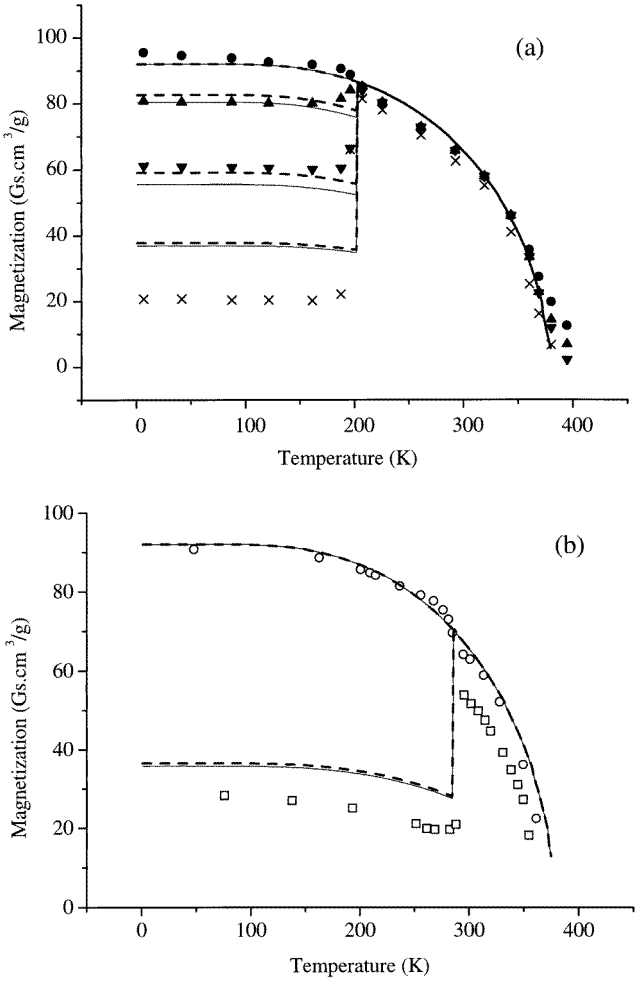

Figure 2. Temperature dependences of the magnetizations of Ni_2MnGa alloys with $T_M = 202$ K (a) and $T_M = 285$ K (b). The theoretical curves correspond to the five-layer periodic structure (solid lines) and 7R structure (dashed lines). The curves are computed for $H_1 = 11.5$ kOe, $H_1 = 1$, 4, 8, and 16 kOe (a) and for $H_1 = 11.5$ kOe, $H = 0.82$ and 15 kOe (b). The experimental data obtained in [1] are represented for comparison by crosses ($H = 1$ kOe), triangles ($H = 4$ and 8 kOe), and solid circles ($H = 16$ kOe). The values measured in [3] are represented by the open squares ($H = 0.82$ kOe) and circles ($H = 15$ kOe).

reasonable, but it should be stressed that the accurate computation of $\langle \boldsymbol{M}(T) \rangle$ dependences cannot be carried out in the framework of the pure phenomenological theory, because both the M_0- and the $\lambda_\perp$-values change abruptly within the temperature range of the martensitic transformation [1, 8, 7]. This effect must be attributed to the change of the quantum electron structure of the crystal during the phase transition.

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